THE SIMPLE DISCHARGE RELATIONS OF SURFACE TENSION AND VISCOSITY ON THE RECTANGULAR FLAT-CRESTED SLIT WEIR

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Abstract

It has been recently shown that a rectangular slit weir can be used effectively as discharge measurement at small flow rates. In this study, slit weir concept is extended to the flat-crested surface to measure the discharges of small flow rates at low weir heights. The purpose of this study is to find out the relations of surface tension and viscosity on the discharge of water flow over a contracted rectangular flat-crested slit weir. This study was carried out on nine different weir heights with 10 mm weir width and 4 mm weir crest thickness. The study shows that the discharge coefficient is dependent on Reynolds number R and Weber number W and the related equations can be used to estimate the discharges of water flow over a contracted rectangular flat-crested slit weir. The relative errors of the coefficient of discharge are good as they are within $\pm$ 5%.

Keywords: discharge coefficient; slit weir; surface tension; viscosity

1.0 INTRODUCTION

The weir has been used for a number of decades in the measurement of discharge in the channel. Rectangular sharp-crested weirs placed perpendicularly across the rectangular channels are widely used for the measurement of flow in the laboratory as well as in small channels in the field (Wu and Rajaratnam, 1996). The fluid properties that influence the discharge over the weir are surface tension and viscosity. The Reynolds number R and Weber number W represent the effect of viscosity and surface tension respectively. However, experiments have shown that the effects of viscosity and surface tension of the liquid only important at low heads and these have not been properly considered in deriving the discharge relations (Raju and Asawa, 1977). Mitra and Mazumdar (2004), found that the accuracy of the relationship of discharge coefficient with viscosity and surface tension for sharp-crested rectangular weir and right-angled triangular weir are within $\pm$ 0.1%.

The extended study for rectangular sharp-crested weir by Gharahjeh et al. (2015) showed that the discharge coefficient changed abruptly in low Reynolds number. Generally, to prevent surface tension and viscosity effects on discharges and to keep the nappe from clinging to the crest, the weir should be sized for measuring at heads more than 60 mm (USBR, 1997). In practice, if sharp-crested weirs are used, they are difficult to be maintained to obtain the discharge for long periods, because the crest is likely to become dulled or rusted or may be damaged by debris. Thus, under this condition, it may be advisable to use a weir with a thicker crest. The narrow rectangular sharp-crested weir known as slit weir, which is capable of measuring small discharges was introduced by Aydin et al. (2002). In this study, the slit weir is extended to weir which is not sharp-crested called rectangular flat-crested slit weir and the related equation is obtained to measure the water discharges.

Assuming ideal conditions, to obtain the actual discharge, a discharge coefficient C_d is introduced to account for simplification and assumptions (Hamil, 2001) so that a simplified form of actual discharge equation Q_A for rectangular sharp-crested can be written as;

$$Q_A = c_d \frac{2}{3} b.H^{3/2} \sqrt{2g.} \qquad (1)$$

where, b = weir width; H = weir head; g = gravitational acceleration

A general dimensional analysis (Ackers et al. 1978) yields that C_d is dependent on weir head H, Reynolds number R (viscous effects) and Weber number W (surface tension effects).

2.0 EXPERIMENTAL SETUP AND PROCEDURES

The experimental setup was constructed to study the form of the relationship between the discharge coefficient and dimensionless numbers, i.e., Reynolds number and Weber number, includes a weir and an approach channel. The slit weirs were fabricated from perspex plates 4 mm thick, which is similar to the weir crest thickness as shown in Figure 1. The approach channel is horizontal, 0.2 m wide, 0.2 m deep, 1.1 m long rectangular, smooth and made of stainless steel. The channel entrance is installed with baffle plates to ensure smooth entry of water in the channel. Thus, it was possible to maintain a disturbance free flow in the channel upstream of the weir.

A flow control valve is installed at the upstream to make fine adjustments to the flow rate. The water in the channel is supplied and circulated using a water pump at a constant speed. The temperature of the water is around 22° C during the experiments. Kinematic viscosity ν =1 x 10^{-6} m^2/s and surface tension σ = 0.0728 N/m of water are taken for analysis. Plasticine is used to prevent water leakage between channel walls and frame edges of weir plate. Measurements were conducted in a channel width B = 0.2 m with nine different weir heights P = 0.035, 0.04, 0.045, 0.05, 0.055, 0.06, 0.065, 0.07 and 0.075 m. According to Aydin et al. (2002), the width of a weir has not classified a slit if the width exceeds more than 0.075 m. Therefore, the weir width b = 0.01 m is used for this study. The minimum and maximum values of weir head H measured for nine weir heights are shown in Table 1. The water flow discharges measured by a Rotameter are 0.000333, 0.000417, 0.0005, 0.000583 and 0.000667 m^3/s.

Ackers et al. (1978) stated that the gauging station should be located sufficiently far upstream to avoid the area of water surface drawdown effects, that is between three to four times the maximum total head over the weir. Thus, in this study, the location is about 3.7 times of the maximum total head over the weir upstream from the face of the weir (i.e. 390 mm from the weir plate). The head on the weir was determined as the average of repeated measurements by a steel ruler for each weir height. The head measurements are repeated for nine different weir plates with the total of 45 runs in which there were five series of discharges for each weir plate.

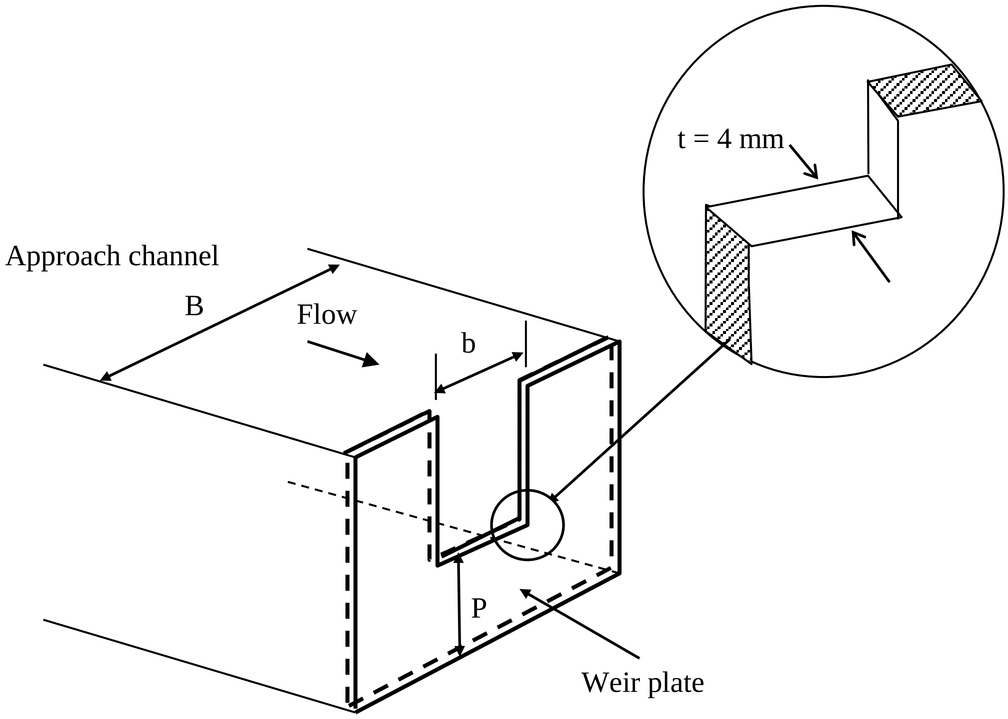

Figure 1 A diagram of contracted rectangular flat-crested slit weir (credited to Jaafar, 2015)

3. DISCUSSION OF RESULTS

The discharge coefficient, C_d is obtained by substituting the actual value of weir head, H, weir width, b and respective discharge, Q_A in equation (1). The variable value of R, W, P, H, actual C_d and predicted C_d are given in Table 1. The result of discharge coefficient, C_d values are plotted against the Reynolds number, R and Weber number, W as shown in Figure 2 and 3 respectively. Both graphs showed a linear relationship between C_d and R and also with W with positive slopes representing a straight line. This indicates that C_d increases with the increase in R, and similar result is shown for W. It means that the data show a clear dependence of C_d on R and W. This is supported by the result obtained by Ramamurthy et al. (2007), in which the C_d of a rectangular multislit weir at a wide range of flow rates is the main function of the R. Similar results were obtained by Aydin et al. (2002, 2006), in which the C_d of a single rectangular sharp-crested slit weir at small discharges is the sole function of R and W.

Table 1 Ranges of variable

Variable	Minimum value	Maximum value
R	4 826.09	6 972.13
W	31.85	66.47
P	0.035 m	0.075 m
H	0.063 m	0.104 m
Actual C_d	0.622	0.763
Predicted C_d [Eq. (2)]	0.654	0.724
Predicted C_d [Eq. (3)]	0.652	0.722

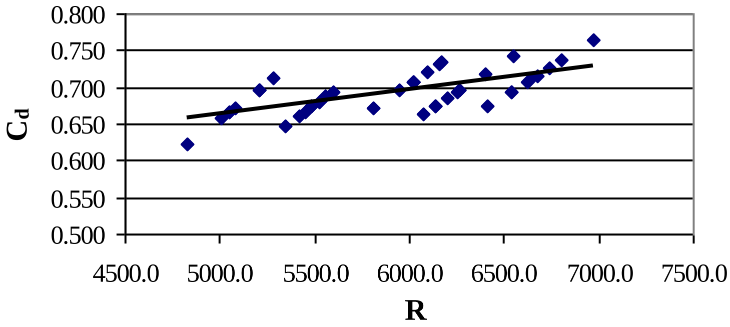

Figure 2 Discharge coefficient versus Reynolds number for rectangular flat-crested slit weir

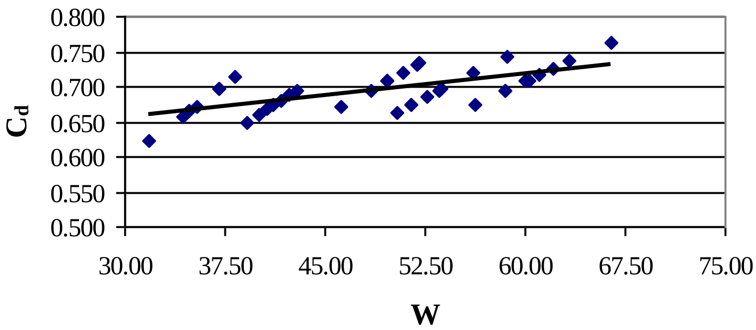

Figure 3 Discharge coefficient versus Weber number for rectangular flat-crested slit weir

According to Ramamurthy et al. (2007), C_d does not depend on large values of R (i.e; > 40 000) in which the value of C_d will constant at 0.61. The C_d of rectangular sharp-crested has the fixed value of 0.7 for Re > 20 000 as obtained by Arvanaghi, H and Oskuei N.N (2013).

The best-fit discharge coefficient equations are found as below:

$$C_d = 0.881 - 1096.1/R \quad (2)$$
$$C_d = 0.786 - 4.28/W \quad (3)$$

Equation (2) and (3) are valid for $0.853 < H/p < 2.852$, $Re < 7\,000$ and $W < 70$ and C_d will be acquired between 0.65 - 0.72. Beyond these boundaries, C_d is not constant and it is not recommended to use a unique C_d for different flow conditions. Actual C_d through a rectangular flat-crested slit weir can be calculated by using the measured head over the weir in equation (1) and predicted value of C_d from the equation (2) or (3). The values of actual C_d were plotted against predicted C_d as shown in Figure 4 and 5. Both graphs showed a linear relationship with a small positive slope representing a straight line. The figures reveal that actual C_d increases with the increase in predicting C_d. It means that the determinations of predicted C_d by using equation (2) and (3) are acceptable.

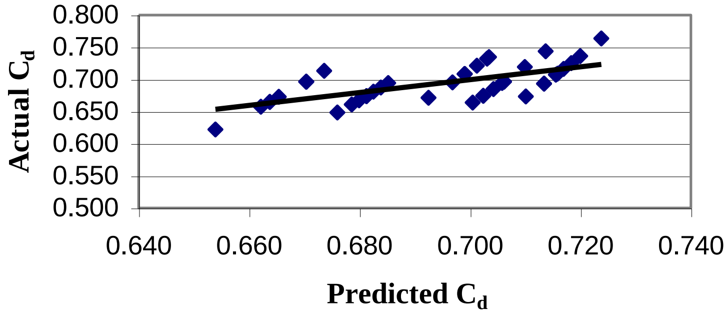

Figure 4 Actual C_d versus predicted C_d (equation 2) for rectangular flat-crested slit weir

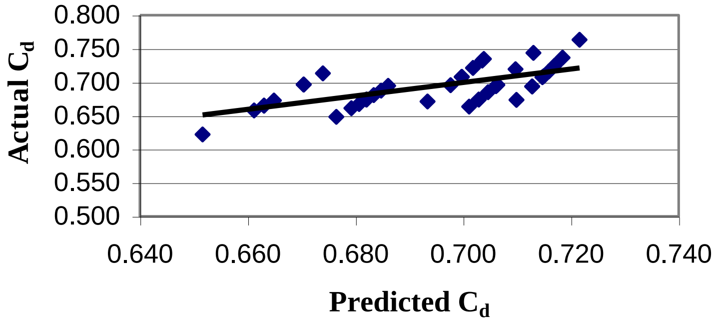

Figure 5 Actual C_d versus predicted C_d (equation 3) for rectangular flat-crested slit weir

The results of the regression analysis are shown in Table 2. The correlation coefficient indicates a moderately strong relationship between the variables. The R-squared values indicate that the model explains 46 % and 45% of the variability of the actual C_d values respectively. The remaining variability is from the experimental error that is mainly due to the measurement of discharges rather than measurement of the head. The relative error is within $\pm$ 5% of the data from both equations and the error is greater than the value obtained by Aydin et al. (2006) which is $\pm$ 2% for rectangular sharp-crested slit weir but it is still quite good.

Table 2 Results of regression analysis

Parameter	Predicted C_d: equation (2)	Predicted C_d: equation (3)
R-squared	0.46	0.45
Correlation coefficient	0.68	0.67
Standard error of estimate	0.02	0.02

Even though the R-squared do not reach more than 70% (low) but the data falls in a symmetrical pattern and have a constant spread throughout the range. This still indicate a real relationship between the significant predictor and the response variable. Furthermore, the R-squared of each weir height is more than 90% for both equations and the standard error of estimate (2%) indicated that the prediction C_d using equation (2) and (3) are precise. In many cases, the standard error of the regression is preferred rather than R-squared (Frost, 2014). In this case, the standard error of estimate is used to accept the proposed equations. Thus, the results of the analysis can confirm that the proposed prediction equations are acceptable.

In this study, the obtained values of R and W are 4 826 to 6 972 and 32 to 66 respectively. The values are considered very small compared to the values studied by Aydin et al. (2002) for rectangular sharp-crest slit weir in which the values of R and W were 2 164 to 42 572 and 13 to 329 respectively. Ramamurthy et al. (2007) in their research for rectangular sharp-crested multislit weir have also studied the bigger values of R and W that ranges between 3 432 to 39 299 and 47 to 1 821 respectively. The result shows that at small R, the inertial forces are low and viscous forces cannot be neglected. Hence C_d will depend on R and W. It is also noted that the effects of surface tension drastically increase when low inertial forces occur. They also reported the effects of Cd on R; however, the trend was different in which C_d decreases as the values of R increases. The difference between the results may be attributed to the fact that in this study the model is not sharp crested and the flow rates were low. These were supported by Montes, 1963 [source: Ackers et al.1978] who stated that the thickness of the crest of the weir plate might have some effect on the C_d which led to the conclusion that the fluid property term was directly related to the crest thickness. The Kindsvater and Carter, 1957 [source: Ackers et al.1978] also reported that fluid property are affected at low flow rates (low Reynolds numbers and Weber numbers). Swamee et.al (2001) found that the terms of viscosity and surface tension could be considered to include in the discharge coefficient equation for sharp-crested rectangular weirs.

4.0 CONCLUSIONS

This paper has presented a successful analysis of discharge coefficient on water free flow over a contracted rectangular flat-crested slit weir. This study shows that surface tension and viscosity of the water effect on the discharge coefficient. Thus, discharge coefficient is a function of R and W. The generated discharge coefficient equations can be used to estimate the water discharges over a contracted rectangular flat-crested slit weir at small discharges and low weir heights in which the relative errors are quite good which is within $\pm$ 5%. It is noted that the equations are for clean water, so the constant values in the relations for C_d may change slightly for different fluids. However, the basic natures of the relations will remain same.

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